Correspondence

Computation of Impedance of Partially Filled and Slotted Coaxial Line

The upper and lower bounds of the characteristic impedance of the slotted coaxial line has already been evaluated by means of a variational expression.[1],[2]

In a slotted coaxial line, filled with an inhomogeneous dielectric as illustrated in Fig. 1, the boundary conditions are complicated and the characteristic impedance, the velocity ratio, and potential distribution are not simply determined.

For the problem presented in this correspondence the relaxation method previously treated by Southwell[3] is very powerful. The characteristic impedance and velocity ratio were obtained by digital computer in this work.[4],[5]

The slotted and partially filled coaxial line as illustrated in Fig. 1 is represented by Laplace's equation in polar coordinates

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} = 0. \quad (1)$$

From a computational aspect, rectangular coordinates (x, y) are more useful than polar coordinates. Under the transformation

$$x = \ln r,$$
$$y = \theta. \quad (2)$$

Laplace's equation (1) takes the following form in rectangular coordinates

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \quad (3)$$

where

- ϕ is the potential,
- a is the radius of the inner conductor in Fig. 1, and
- b is the radius of the outer conductor in Fig. 1.

After transforming the coordinate system the problem of a partially filled and slotted coaxial line becomes a problem of a periodic microstrip whose period is 2π.

From symmetry, it is sufficient for the purposes of computation to consider the part of the x, y planes as illustrated in Fig. 2. The numerical method for solving the relaxation equation was successive overrelaxation and the scanning of the mesh was done by consecutively progressing up the mesh by traversing rows from left to right. The total number of mesh points was about 700 and number of relaxations of each point was about 600. Computation was done for four different values of ratio b/a, i.e., $\ln b/a=0.833$, 1.0, 1.25, and 1.877 which corresponds to closed air-filled coaxial lines of 50, 60, 75, and 112.6 ohms of impedance, respectively. For each $\ln b/a$, three different values of the dielectric constant ϵ_2 were tried, i.e., $\epsilon_2=1.00$ (homogeneous case), $\epsilon_2=2.26$, and 5.00 (inhomogeneous case). The dielectric constant ϵ_1 in Fig. 1 is always 1.00, i.e., air.

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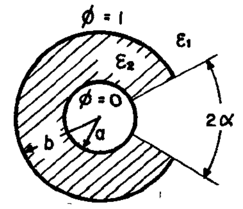

Fig. 1. Partially filled and slotted coaxial line.

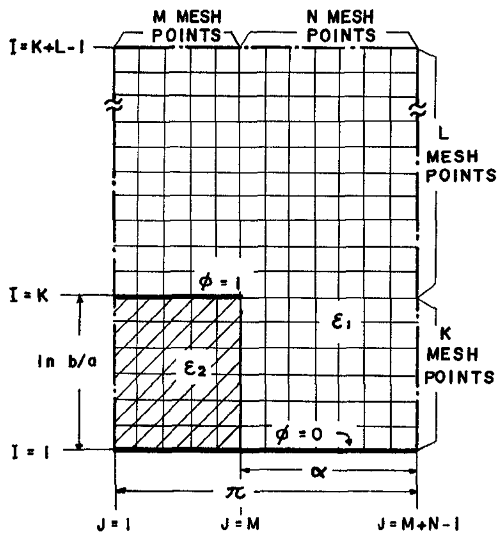

Fig. 2. Network to be computed.

TABLE I
CHARACTERISTIC IMPEDANCE FOR SOME VALUES OF L

L	Z (ohms)
	130.6456
1.25K	127.61619
2K	125.00116
2.5K	124.5427
3K	124.40304
4K	124.32919
4.5K	124.32099

The choice of L affects the result as follows. In one typical case of $\ln b/a=1.00$, $\epsilon_2=1.00$ and $\alpha=147°16'$, the characteristic impedance corresponding to some values of L changes is shown in Table I.

From Table I, the value of L was selected to be three or four times larger than the value of K.

The first step in computation is to obtain the potential at each mesh point. The charges on the conductors may be found by Gauss' theorem, and the field gradients of this theorem obtained by linear interpolation.

The capacitance was then obtained from

$$C = \frac{Q}{V}$$

where Q is the charge and V is the potential difference between the inner and outer conductors; the characteristic impedance and the velocity ratio were also obtained.

Figs. 3 through 6 show the characteristic impedance and velocity ratio for $\ln b/a=0.833$, 1.00, 1.25, and 1.877, respectively. The case where $\epsilon_2=1.00$ is the solution of an homogeneous line; Duncan and Minerva[2] have given the characteristic impedance for this case. Accuracy of the result by the relaxation method is within 0.5 percent.

The significance of the work presented here is that the solution of inhomogeneous cases are obtained. In these cases the boundary conditions imposed by both conductors and inhomogeneous dielectric media are complicated to the degree that an analytic solution of this type is very difficult.

Corroborative experimental results appear as plotted points in Figs. 5 and 6.

The experiment was done by measuring the capacity of an inhomogeneous coaxial line about 0.5 meters in length which was partially filled with polyethylene ($\epsilon_2=2.26$) and slotted with a certain open angle.

To avoid end effects a longer line was used so the averaged value was not as influenced by these effects. Also, to avoid the finite wall thickness of the outer tubing, very thin outer tubing was used.

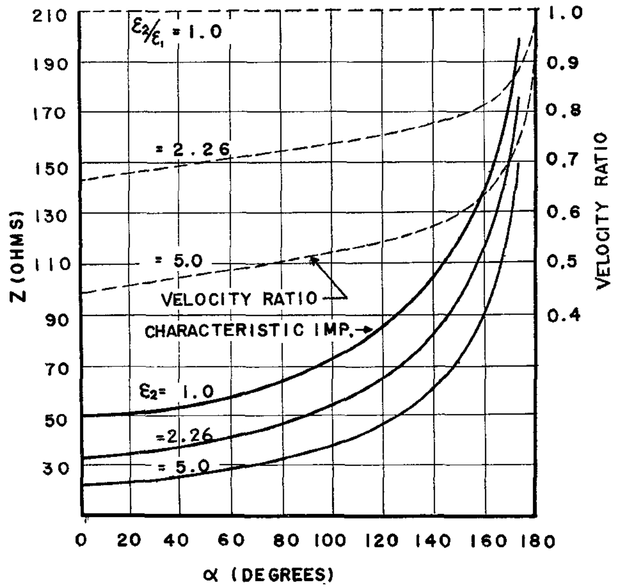

Fig. 3. Characteristic impedance and velocity ratio of partially filled and slotted coaxial line for ln $b/a = 0.833$.

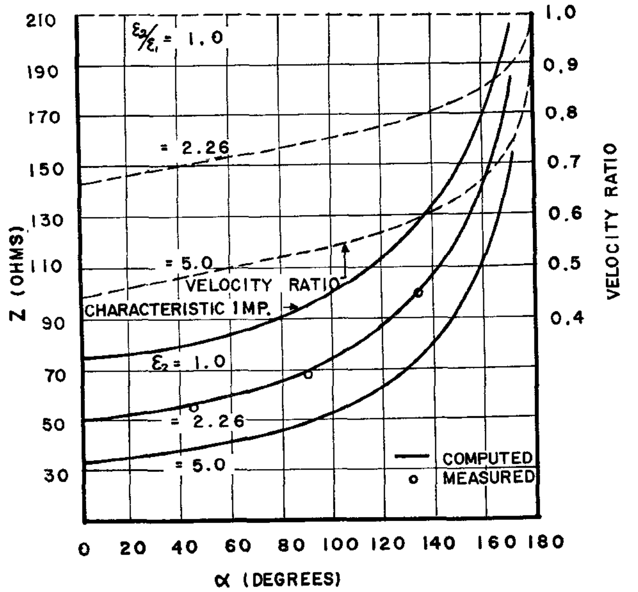

Fig. 5. Characteristic impedance and velocity ratio of partially filled and slotted coaxial line for ln $b/a = 1.25$.

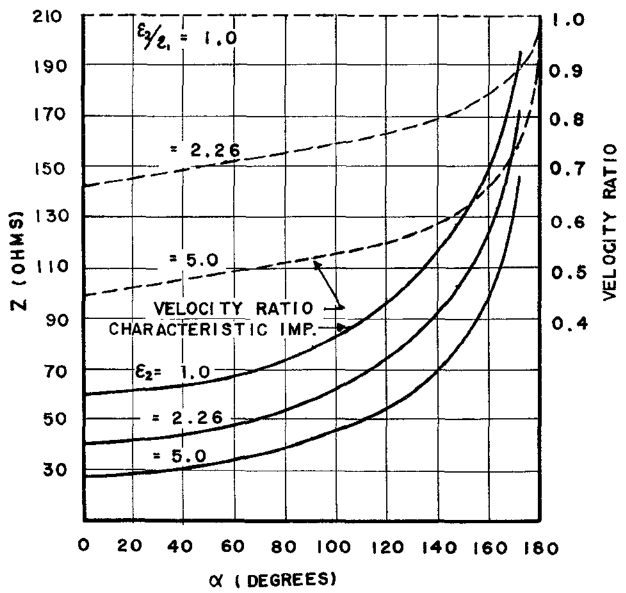

Fig. 4. Characteristic impedance and velocity ratio of partially filled and slotted coaxial line for ln $b/a = 1.00$.

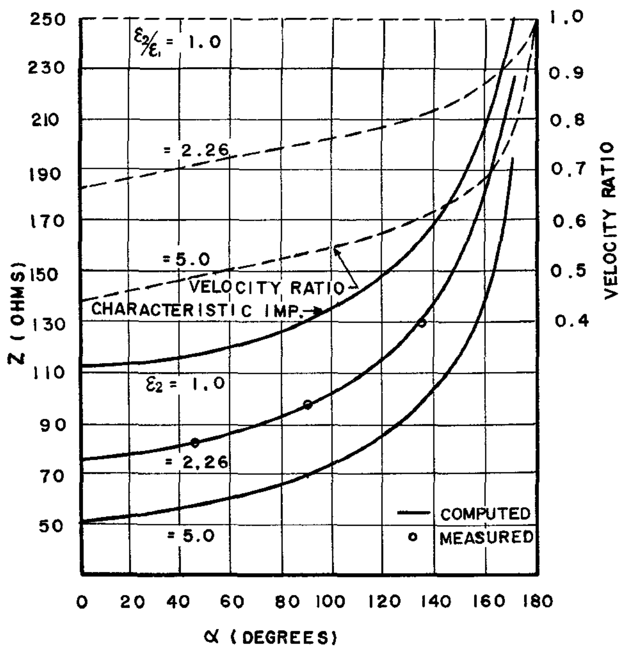

Fig. 6. Characteristic impedance and velocity ratio of partially filled and slotted coaxial line for ln $b/a = 1.877$.

Three coaxial lines with differing angles, i.e., 45, 90, and 135 degrees, were prepared and measured. The measurement of capacitance was done by using a Q meter which covered frequency ranges between 100 kHz and 10 MHz.

The characteristic impedance and the velocity ratio of slotted coaxial lines were obtained by this work. The result of inhomogeneous cases can be used for the design of a partially filled balun which is a wideband transformer from a closed and filled coaxial line to parallel line or microstrip filters and other special transmission lines.

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